SASC Guidance on assessment of Dyscalculia and Maths Difficulties within other Specific Learning Difficulties

This guidance has been produced by the SASC/STEC Dyscalculia Working Group, which was set up following a SASC/STEC Consultation Day at which a panel of experts in assessment and teaching of mathematics (Clare Trott, Carla Finesilver, Steve Chinn, David Grant and Pete Jarrett) discussed issues relating to the diagnostic assessment of Dyscalculia and Maths Learning Difficulties

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Maths Learning Difficulties is an umbrella term used in the United Kingdom to describe problems with learning and applying mathematical facts and procedures. Over the past 20 years there has been a considerable increase in research into the underlying causes of mathematics learning difficulties, including many studies of the nature and causes of mathematics difficulties within dyscalculia and other specific learning difficulties (SpLDS). The emerging evidence is at times contradictory, and there are still many questions to be answered, but there is international consensus on certain subjects, including how to differentiate between dyscalculia and mathematics difficulties within other SpLDs.

The aim of this new SASC guidance on dyscalculia, SpLDs and maths learning difficulties is to provide assessors with updated, evidence based, operationally effective definitions and procedures which will enable them to differentiate between three categories of mathematics learning difficulties:

1. Dyscalculia – a SpLD whose core feature is a problem with sense of number
2. Other SpLDs which do not include a problem with sense of number, but which may have an impact upon mathematics learning
3. Maths learning difficulties arising from lack of appropriate teaching, environmental factors or other medical conditions**.**

Key Principles

The following 4 principles apply to **dyscalculia** and **other SpLDs** and differentiate them from other mathematics difficulties.

1. Difficulties must be unexpected in relation to age, level of education, level of experience and level of other attainments:

2. Difficulties should be specific and persistent.

3. Difficulties must not be solely caused by other factors such as:
    - Inappropriate teaching or gaps in mathematics education
    - Social and personal factors which adversely affect attitude/motivation with regard to learning mathematics
    - Maths anxiety
    - Incomplete mastery of the language of instruction (e.g. EAL/ESL)
    - General learning difficulties

4. Difficulties should not arise from another neurological, physical or mental health condition.

In addition to the above, the following principles have been adopted to differentiate between dyscalculia and other SpLDs:

1. Dyscalculia

The core feature of dyscalculia is a domain specific deficit in sense of number. This manifests in many ways, some of the key ones being difficulties with subitising, symbolic and non-symbolic magnitude comparison, and ordering (cardinality, ordinality). People with dyscalculia will also have a wide range of other mathematics difficulties: understanding of number and numeric relationships is essential to the development of skills in estimation, manipulation of quantities and arithmetic. Arithmetic is the first stage of mathematics teaching, so difficulties in this area are likely to have a negative impact upon subsequent mathematics learning. Dyscalculia can co-occur with other SpLDs.

2. Other Specific Learning Difficulties

Other SpLDs do not include a deficit in sense of number, but are underpinned by domain general deficits which can include language, memory, planning and sequencing, processing speed, attention, perceptual reasoning, visual-spatial skills and/or motor coordination. There is substantial evidence that these general domain deficits can potentially affect all types of learning, including mathematics.

There are concerns within the mathematics community that problems with mathematics arising from SpLDs are often less well understood than co-occurring literacy difficulties. This is partly because there has been far more research into dyslexia (and in particular, the effect of dyslexia upon literacy) than any of the other SpLDs.

It is important to redress this imbalance by ensuring that difficulties with mathematics are explored appropriately within diagnostic assessments. Where those difficulties are found to have a greater impact than difficulties with literacy, assessors could note that *the specific learning difficulty has a clear and specific impact upon mathematics.*

However, it should not be assumed that general domain deficits will inevitably lead to weaker performance in mathematics. As with literacy, appropriate training, extensive practice, and compensatory use of other strengths and strategies can enable an individual to perform well in mathematics despite domain general or domain specific deficits.

Dyscalculia Definition

Dyscalculia is a specific and persistent difficulty in understanding numbers which can lead to a diverse range of difficulties with mathematics. It will be unexpected in relation to age, level of education and experience and occurs across all ages and abilities.

Mathematics difficulties are best thought of as a continuum, not a distinct category, and they have many causal factors. Dyscalculia falls at one end of the spectrum and will be distinguishable from other mathematics issues due to the severity of difficulties with number sense, including subitising, symbolic and non-symbolic magnitude comparison, and ordering. It can occur singly but can also co-occur with other specific learning difficulties, mathematics anxiety and medical conditions.

What should be included in an assessment of difficulties with mathematics?

Diagnostic assessment of difficulties in mathematics should form part of a holistic assessment designed to explore the full range of SpLDs and cognitive, medical and environmental factors that may be contributing to difficulties with learning. As such they should include the following:

1. A framework for a thorough and appropriate history taking which covers mathematics, literacy and wider barriers to learning.

2. Tests of verbal, visual and visual-spatial reasoning and cognitive processing (such as memory, phonological processing, processing speed and accuracy, visual spatial sequential skills) in order to identify domain general strengths and weaknesses within the cognitive profile.

3. Tests of literacy and mathematics skills in order to identify strengths and weaknesses within the attainment profile.

4. Informal, qualitative tests of understanding of number that use subitising, symbolic and non-symbolic magnitude comparison, ordering and concrete tools to explore concept of number. This could include screeners designed to explore number sense.

5. Standardised measures of
   - Arithmetic (+, -, x, ÷). Timed and untimed to establish what difference time pressure makes upon performance,
   - Mathematics reasoning and problem solving, including word problems to explore whether the difficulties are related to number, or mathematical terminology, or language more generally.

6. Qualitative analysis of performance within these tests:
   - Analysis of the individual's pattern of errors,
   - Observation and questioning about strategies used,
   - Observation of motivation, determination, perseverance, impulse inhibition, attention, and which tasks were avoided,
   - Conceptual understanding of any standard procedures used,
   - Use of concrete materials and visual representations – to evaluate to what extent an individual understands basic mathematical concepts, and to explore any differences between what an individual can achieve with standard symbolic notation, and with nonstandard representational strategies (e.g. modelling, drawing).

7. Recommendations for interventions and reasonable adjustments should be clearly linked to:

- The individual’s difficulties reported in the background information, and evidenced in the assessor’s quantitative and/or qualitative analysis of performance in tests.
- The individual’s needs within the classroom, course or job. Wherever possible recommendations should be developed collaboratively with relevant mathematics specialists in the individual’s school, course or workplace.

Reasonable adjustments should be appropriately targeted to address the need without potentially giving the individual an unfair advantage. Assessors should bear in mind that adjustments such as use of a calculator or provision of rest breaks can sometimes target the need more effectively than additional time.

Who can assess for difficulties with mathematics and what training/ qualifications will they need?

It will be evident from the above guidance that difficulties in mathematics should not be assessed and evaluated purely on the basis of a score derived from one test of arithmetic skills (in the same way that literacy skills would not be assessed purely on the basis of a score derived from one test of single word reading).

Assessors should have the knowledge and expertise in mathematics that is necessary to be able to perform the range of investigations listed in "***What should be included in an assessment of difficulties with mathematics?"*** *(*see above). This does **not** mean that one needs to have a degree in mathematics to assess mathematics, but in order to assess mathematics skills within a diagnostic assessment, assessors should have **knowledge and training from both a) and b) categories below:**

a) Knowledge and experience of holistic diagnostic assessment

- Training in, and knowledge and experience of the process of performing a holistic diagnostic assessment which synthesises and evaluates qualitative and quantitative evidence gathered from detailed history-taking, psychometric testing, observation and error analysis.
- Training in, and/or experience of, applying this knowledge to the assessment of mathematics.

This knowledge and experience can be acquired through:

- A Level 7 qualification which explicitly trains and assesses the assessor in the full process of diagnostic assessment, including extensive coverage of diagnosing difficulties with mathematics or mathematical cognition (for example a Masters in Psychology, or a Level 7/PG Dip qualification in SpLD Assessment).
- If the assessor's **original Level 7 assessment qualification** did not cover mathematics explicitly and extensively, it is expected that a top-up Level 7 course in mathematics and dyscalculia will be undertaken (60 credits plus 20 hours experience of mathematics teaching) **unless** the assessor has acquired the required skills and knowledge through extensive CPD training, mentoring, personal research and experience and can demonstrate competence in applying that knowledge to diagnostic assessments of mathematics or mathematical cognition.

b) <u>Knowledge of mathematical skills development including:</u>

- The impact that domain specific and domain general deficits may have upon learning and performance in mathematics in education, the workplace and everyday life.
- The range of strategies and procedures an individual might use to perform calculations and solve mathematical problems at the individual's current level of mathematics experience and training. An understanding of which strategies and procedures are least/most efficient and effective in different situations.
- The stages and processes by which mathematics skills are developed and the normal range of variation that might be expected for the individual's age and/or level of mathematics education.
- Typical error patterns for individuals who are struggling with particular aspects of mathematics.
- The impact that mathematics anxiety may have upon learning and performance in mathematics.

This knowledge can be acquired through:

- Experience of teaching mathematics skills (for example: as a Primary Teacher, a QTS teacher, SpLD Tutor, Basic/Functional Skills Tutor, Mathematics Teacher);
- **And/or** a Bachelor Degree or Postgraduate qualification which explicitly and extensively covers how maths skills and/or maths cognition develop (for example: a BEd or PGCE in a mathematics related subject; a BA/BSc or Masters in Psychology which includes extensive coverage of mathematics and/or mathematical cognition; a postgraduate qualification in SpLD Tutoring which includes extensive coverage of maths difficulties within SpLDs) and subsequent professional practice and CPD.

It is a requirement of HCPC and the BPS, and APCs and their relevant governing bodies that assessors should work within their range of expertise (acquired through appropriate qualifications and experience), use appropriate assessment tools, and be fully up to date in their professional development. Any diagnostic decision might, ultimately, need to be defensible in court. Where an individual's mathematics difficulties fall beyond the scope of an assessor's professional boundaries, that individual should be referred on for further assessment by a suitably qualified Assessor. This is most likely to occur when:

  - an assessor has only been trained in literacy skills (or has minimal training in mathematics),
  - and/or where dyscalculia (as defined in this guidance) is suspected and the assessor does not have sufficient experience, training and knowledge to evaluate sense of number,
- and/or where the mathematics difficulties are having a very major impact upon the individual's ability to perform effectively in education, the workplace and/or daily life and the assessor does not have sufficient experience, training and knowledge to understand or assess that impact or make suitable recommendations.

Full Version

Introduction

Maths Learning Difficulties is an umbrella term used in the United Kingdom to describe problems with learning and applying mathematical facts and procedures. Over the past 20 years there has been a considerable increase in research into the underlying causes of mathematics learning difficulties, including many studies of the nature and causes of mathematics difficulties within dyscalculia and other specific learning difficulties (SpLDs). The emerging evidence is at times contradictory, and there are still many questions to be answered, but there is international consensus on certain subjects, including how to differentiate between dyscalculia and mathematics difficulties within other SpLDs.

The aim of this new SASC guidance on dyscalculia, SpLDs and maths difficulties is to provide assessors with updated, evidence based, operationally effective definitions and procedures which incorporate the latest developments in the dyscalculia research and which will enable assessors to differentiate between three categories of mathematics learning difficulties:

- Dyscalculia – a SpLD whose core feature is a problem with sense of number,
- Other SpLDs which do not include a problem with sense of number, but which may have an impact upon mathematics learning,
- Maths learning difficulties arising from lack of appropriate teaching, environmental factors or other medical conditions.

Key Principles

There is a degree of international consensus over the criteria for defining SpLDs, although the umbrella term used differs: the American Psychiatric Association Diagnostic and Statistical Manual of Mental Disorders (5th Edition DSM-5) classifies them as Specific Learning Disorders (APA 2013); the International Classification of Diseases, (Eleventh Edition, ICD-11) refers to them as Neurodevelopmental Disorders (WHO 2018); and here in the United Kingdom the preferred term is specific learning difficulties, or specific learning differences.

The following 4 principles broadly reflect the criteria found in DSM-5 and ICD-11 for identifying Specific Learning Disorders/Developmental Disorders of Scholastic Skills and they provide a good basis for differentiating between dyscalculia and other SpLDs on the one hand, and on the other the range of other medical and/or external factors that can affect mathematical learning.

1. **Difficulties must be unexpected in relation to age, level of education, level of experience and level of other attainments**:

   Age: Difficulties should be unexpected in relation to the skills that would normally be expected given the individual's age (WHO 2018, APA 2013). However, since mathematics is developmental in nature, and individuals develop at different rates (Piaget, 1952, Kaufman & Nuerk 2005, Dowker 2005, Szucs & Goswami 2013, Gilmore et al. 2018,), it is important to take normal levels of individual variation into account when assessing the difficulties experienced (Kaufman et al 2013, Butterworth, 2018) especially when working with children. It follows that assessors should have a detailed knowledge of typical mathematical developmental trajectories and the extent of variation that exists within the relevant age range.

   Level of education: Difficulties should be unexpected in relation to the normal range of mathematics achievement expected at the individual's level of education (WHO 2018). This will be dependent on how far the individual has progressed in mathematics studies (primary, secondary, GCSE, A Level, undergraduate and beyond) or the mathematical requirements of the workplace. The University of Cambridge's NRICH website maps out expected mathematical skills development within the National Curriculum (Department for Education, 2014) in the UK for each stage of education from early years to A Level (NRICH no date).

   However, in an increasingly multi-cultural population, many individuals may have received some or all of their education in other countries which may influence mathematical development (Dowker, 2019). A cross cultural investigation by

Rodic et al (2018) found that educational culture has a clear impact on performance in mathematics in different countries. The study revealed that 6–9 year-old Chinese children outperformed Russian and UK children in simple arithmetic and mathematical reasoning tasks despite having broadly similar pre-school mathematical cognitive skills. It attributed this to China's formal education system and greater input from parents who are themselves better able to assist as their own arithmetic and mathematical reasoning skills are relatively strong.

Level of experience: Limited experience of mathematics will have an impact upon performance as development and retention of mathematical skills requires explicit teaching and frequent practice (Sharma, 2016). Factors that can negatively impact on the level of experience may include: gaps in education; poor attendance (Parke & Kanyong 2012, Catton et al 2017); being educated in a different educational culture which focuses on different mathematics skills (Rodic et al 2018); limited practice due to slow performance in class, lack of perseverance, failure to complete homework, etc.; skills fade in adults who no longer use the full range of mathematical procedures and knowledge.

Equally, extensive practice (due to additional teaching, or frequent use of particular mathematical skills in the workplace) can mask underlying difficulties.

Level of other attainments: Relatively few children and adults have difficulties in **both** mathematics and literacy. For example, Parsons & Bryner (2005) found that only 14% of a UK adult cohort study had both poor numeracy **and** poor literacy. Mathematics attainment which is lower than literacy attainment is therefore particularly noteworthy.

2. **Difficulties should be specific and persistent:**

Specific: There will be an uneven pattern of strengths and weaknesses across the cognitive and attainment profiles with areas that are impaired and areas that are "spared" (Flanagan & Alfonso 2015). There can also be specific weaknesses within the mathematics profile – areas of strength alongside unexpected weaknesses (Kaufman et al 2013).

*Persistent***:** The difficulties will be developmental in nature (WHO 2018) and have begun in childhood and persisted over time (WHO 2018 and APA 2013) despite any appropriately targeted interventions (APA 2013). This concept of persisting difficulties seeks to differentiate between individuals who have temporary difficulties (due to short-term developmental delays or gaps in education, etc.) and those whose difficulties are neurological and therefore long-term. A systematic review by Nelson & Powell (2018) of longitudinal studies of maths development, found that early maths difficulties persist over time – although children with early maths difficulties can improve over time, their maths skills remain weak relative to their peers.

Persistence, therefore, is not synonymous with inability to improve. Chinn (2017) emphasises that persistence does not necessarily mean a permanent inability to do something. Dyscalculia is not the same as acalculia – a complete loss of the ability to work with numbers (caused by brain damage). As with other areas of difficulties in mathematics, appropriately targeted interventions can improve performance. Much has been written about effective intervention techniques (see Dowker, 2019. for a comprehensive review). For example; Kaufman and von Aster (2012) note that repeated practice, segmentation of subject matter, and teaching in small, interactive groups, are particularly effective; Alibali et al (2018) found that the relatively simple strategy of colour coding certain parts of an equation enabled young children to improve their problem solving strategies, and Finesilver (2014) has explored the value of using visuospatial representational strategies tailored to learners' specific areas of conceptual and procedural difficulty.

It should not be assumed, however, that failure to respond to intervention inevitably indicates a persistent mathematics difficulty. Watson (2006) demonstrates that well-meant but poorly designed teaching strategies and approaches can in practice exacerbate underachievement in mathematics by making inappropriate demands on learners. Also, it is evident that interventions are most effective when a child is first learning mathematics (Dowker 2005). There is significant literature around the evaluation of the effectiveness of interventions and what ‘good’ interventions may look like (e.g. Kroesergen et al 2003, Carbonneau et al 2013, Dowker 2008, Dowker 2010, Dowker, 2019). Where possible, it is useful to investigate at what stage the individual received help, and what type of strategies were covered.

Persistence is not necessarily synonymous with early presentation. Szucs and Goswami (2013) note that mathematics calls upon different cognitive abilities at different stages and that difficulties may present at later stages when a particular cognitive ability becomes important. The DSM-5 notes that, although there will be some evidence of difficulties during school-age, in some people these will not present as significant problems until adulthood, if academic, work and day-to-day demands become greater (APA 2013). In addition, mathematics difficulties are often not identified in a large number of learners who find mathematics learning ‘hard’ (Morsanyi et al, 2018).

3. **Difficulties must not be solely caused by other factors such as**:

   Inappropriate teaching or gaps in mathematics education. Scholastic skills need to be taught and learned. Inappropriate teaching, or gaps in mathematics education can therefore have a negative effect on mathematics development (Shalev and Gross-Tsur, 2001; Watson 2006, APA, 2013; Desoete, 2015) and difficulties with mathematics must not be due to lack of availability of education (WHO 2018).

   Social and personal factors which adversely affect attitude/motivation with regard to learning mathematics. Data from the PISA 2012 study indicates that low performers in mathematics often have a negative attitude towards mathematics and are less likely to persevere with difficult mathematics problems or to complete mathematics homework (Sizmur & Berge 2015).

   Maths anxiety: there has been considerable research on the impact that anxiety can have upon mathematics._ Anxiety is a complex issue. It can be divided into Trait Anxiety (specific to certain situations or tasks – e.g. maths anxiety) and State Anxiety (general and long-standing) (Hill et al 2016). Either can be present in an assessment.

   Maths anxiety is widespread in the UK and can range from mild to severe. In its most severe form, it can be debilitating (Moore & Ashcraft, 2013). Maths anxiety and mathematics difficulties are interrelated – one feeds the other (Sharma 2016, Boaler 2016) but it is not always the case that individuals with anxiety have difficulties with mathematics, or that people who have difficulties with mathematics are made anxious by it (Brown et al, 2008). Nonetheless, maths anxiety tends to impair skill development and when it is severe it seems to impair basic numerical processing (Kaufman & von Aster 2012). Anxiety may be related to earlier experiences in the mathematics classroom and may have been preceded by perceived 'failures' in learning number concepts (Ma & Xu 2003; Petronzi et al 2019). A recent joint study by the University of Cambridge and Nuffield Foundation provides an excellent overview of the causes and nature of mathematics anxiety and its impact upon mathematics learning (Carey et al 2019).

   One can be very good at mathematics but slow at finding the answers (Mason, 2010). This difficulty is exacerbated in situations where learners are given unnecessary time constraints. The UK education system's focus (in mathematics training and examinations) on speed of calculation can lead to individuals who are capable of solving problems accurately being labelled as having a difficulty with mathematics. This can increase mathematics anxiety (Chinn & Ashcraft 2015, Boaler 2016).

Anxiety is known to have a negative impact upon working memory (Moore, McAuley, Allred & Ashcraft 2015, Kaufman & von Aster 2012). Students who rely on procedural learning (rather than a deeper understanding) are more at risk of their anxiety affecting performance, because working memory is central to success in procedures (Boaler 2016). Strategies that enable the student to reason from known facts and to make connections between concrete, symbolic and visual representations of knowledge through language can reduce the load on working memory and provide a fall back when difficulties arise (Leong et al 2015).

It follows, therefore, that an assessment of mathematics difficulties should include some exploration of both general and trait anxiety, with particular sensitivity to the learner's self-concept as a mathematics learner. This should include exploration of the conditions under which the learner feels anxious, the learner's beliefs about themselves as mathematics learner, and the impact anxiety has upon the individual's studies (such as avoidance of mathematics tasks, or the subject choices they make) (Boaler, 2016; Chinn & Ashcroft 2017, Chinn 2017, Carey et al 2019).

Incomplete mastery of the language of instruction. DSM-5 requires that difficulties with learning should not be caused by problems with speaking or understanding the language (APA 2013). This reflects a wealth of evidence that lack of fluency in the language of instruction will have a negative impact upon learning.

General Learning Difficulties DSM-5 and ICD-11 both require that difficulties should not be due to intellectual disability (APA 2013) or general mental retardation (WHO 2018) – in both cases defined as IQ scores below 70.

4. **Difficulties should not arise from another neurological, physical or mental health condition**.

Both the DSM-5 and ICD-11 (APA 2013, WHO 2018) require that other neurological, physical or mental health conditions be excluded as the sole cause of learning difficulties before diagnosing a SpLD.

Differentiating between Dyscalculia and mathematics difficulties within other Specific Learning Difficulties

The level of consensus that exists about the broad criteria for specific learning/developmental disorders/difficulties/differences begins to break down when trying to define the cognitive and behavioural symptoms that will be present in relation to mathematics. ICD-11 and DSM-5 both broadly include problems with number sense, memorizing math facts, math calculations, math reasoning and math problem solving (APA 2013) but there is considerable wider debate about whether all of these features should be included within the term “dyscalculia”.

Mazzoco and Rasanen (2013) identify a range of factors which contribute to the lack of consensus. Amongst other things they note that mathematics is developmental in nature (i.e. it involves the gradual growth of abilities, skills and knowledge over time, from birth onwards) and involves a very broad range of cognitive abilities, skills, knowledge and strategies. There will, therefore, be wide variation in mathematics skills within any population or any age group, and there is currently limited knowledge as to what represents a “typical” developmental trajectory for mathematics. This creates problems in establishing boundaries between “typical” development on the one hand, and impairments/difficulties/ differences on the other.

Secondly, mastery of mathematical skills is influenced by complex interactions between innate, heritable features and environmental factors. Thus, two children might have apparently similar gaps in skills, but the underlying causes of those gaps could be completely different. Zhang et al (2017) found that children with early deficits in domain-general cognitive skills such as visual-spatial perception, language, rapid automatized naming (RAN), and working memory may experience difficulty learning and understanding numbers. Szucs & Goswami (2013) draw attention to a cascade effect whereby suboptimal learning in one area of mathematics can lead to a cascade of mathematics failures over time. Cowan et al (2018) explore the reciprocal relationship between general cognitive abilities and mathematics learning – demonstrating that mathematics learning contributes to development of cognitive abilities and vice versa. These interrelationships make it very difficult to tease out cause and effect or to delineate distinctive pathways of mathematics failure.

Thirdly, research on neurodevelopmental difficulties in mathematics is limited compared to research on other SpLDs (Bishop, 2010) and the knowledge base on which definitions can be founded is still emerging (Mazzoco and Rasanen 2013). Ansari and Bugden (2014) note that studies of brain activity during numerical magnitude processing and arithmetic activities reveal that many brain regions function differently between typically developing children and those with developmental dyscalculia which makes it difficult to pinpoint a single neurological cause of dyscalculia. Berch et al (2015) also note that the study of the brain requires synthesis of quantitative and qualitative evaluation of complex environmental, genetic and neurological interactions – and that this can sit uncomfortably for experts trained in scientific research. The consensus seems to be that progress is being made – the level and quantity of research into mathematics difficulties is accelerating – but that there is still much to be learned.

Another difficulty with arriving at agreed definitions is the ongoing debate about domain general versus domain specific deficits. Kaufman et al (2013) summarise this debate in their article which sets out to achieve some level of international consensus on developmental dyscalculia. They note that difficulties in arithmetic can arise from deficits in cognitive domains (such as working memory, processing speed, attention, visual perceptual reasoning) which are general as they impact upon all aspects of learning. If one defines dyscalculia as a difficulty with arithmetic or computational skills, then this raises the question of how to differentiate between dyscalculia, other SpLDs, and other mathematical difficulties arising from external or medical factors. An alternative is to focus on a cognitive domain which is specific only to mathematics – i.e. numerosity or the sense of number. Adopting a difficulty with sense of number as the core feature of dyscalculia has the advantage that it makes dyscalculia distinct from other SpLDs and clearly identifiable in an assessment.

Domain specific sense of number

Our understanding of the cognitive basis of number is developing rapidly (Leibovich et al 2017). In recent years research has provided increasing evidence of the existence of a domain specific ability to perceive and manipulate discrete quantities – i.e. a sense of number (Henik et al 2017, Sharma 2015, Karagiannakis et al 2014, Butterworth and Laurillard 2010, Kaufman et al 2013, Landerl et al 2013). A deficit in this sense of number can have a severe effect on the ability to develop arithmetic skills, but it is distinct from working memory – individuals with severe difficulties with numerosity often do not have a deficit in working memory (Butterworth & Laurillard 2010). It would seem that development of this sense of number is partly dependent upon an innate ability to perceive and evaluate continuous, non-countable dimensions such as size, area, brightness, density, which begins to develop in the first months of life and which is closely related to visual-spatial skills (Henik et al 2017).

The sense of number can be broken down into a range of separate functions, many of which can be measured (Kaufman et al 2013, Butterworth and Laurillard 2010). Definitions of number sense vary and in a review of literature on the topic, Chinn

(2017) refers to literature (Berch, 2005) which identifies as many as 30 components of number sense. Back (2014) helpfully summarises them into 7 common threads:

- An awareness of the relationship between number and quantity,
- An understanding of number symbols, vocabulary and meaning,
- The ability to engage in systematic counting, including notions of cardinality and ordinality,
- An awareness of magnitude and comparisons between different magnitudes,
- An understanding of different representations of number,
- Competence with simple mathematical operations,
- An awareness of number patterns including recognising missing numbers.

For the purposes of diagnostic assessment, four areas are commonly listed as being crucial in identifying a deficit in sense of number.

- _Subitising_ – the ability to rapidly and accurately recognise the number of objects in a small group without having to mentally or physically count them. *Perceptual subitising* is the ability to instantaneously recognise groups of up to 5 items. Above this number most individuals need *conceptual subitising* which utilises additional mental strategies (such as breaking items into smaller groups). For example, a random array of seven dots would not be instantly recognisable as 7, but they would become so if the dots were arranged into two rows of three and a single dot. Subitising helps children to relate abstract numbers to real quantities of objects (number conservation); to develop an understanding that numbers are made up of smaller numbers (the part-whole relationship); and thus to visualise numbers and learn number facts which form the basis of addition and subtraction (Gifford 2018).

  Ability to subitise is commonly assessed through dot arrays.

- _Non-Symbolic Magnitude Comparison_ – the ability to compare objects or groups of objects, to recognise differences in size or quantity and identify which is greater or lesser. For example, a piano is larger than a violin, and a bag containing 5 apples has more than another bag containing 4. Non-symbolic magnitude comparison is commonly measured using dot arrays, or pictures of pairs or groups of objects.

- *Symbolic Magnitude Comparison* – the ability to compare symbols which represent quantities (such as Arabic numerals) to recognise differences in quantity and identify which is greater or lesser. This can be single digit numbers or compositional numbers (multi-digit whole numbers, fractions, decimals).

  There is some argument over whether understanding of non-symbolic and symbolic magnitude develop separately or simultaneously and whether they form part of one concept (Approximate Number System) or are two distinct concepts (Matejko & Ansari 2016, Liebovich et al, 2017, Li et al 2018). Non-Symbolic Magnitude Comparison ability is commonly assessed through tasks which require the individual to identify which number/fraction/decimal is greater or lesser. For both symbolic and non-symbolic magnitude, the speed at which differences can be identified tends to increase as the distance between the quantities or numbers becomes greater (De Wolff et al 2014).

- *Ordering (cardinal and ordinal)* – Cardinality relates to "how many" items there are in a set with similar properties (e.g. 5 applies and 4 pears, or 9 pieces of fruit) whilst ordinality relates to the position of an item or number relative to other items or numbers within a series (Attout & Majerus 2018). For example, the number five is cardinal as it is a symbolic representation of "how many" – a set of 5 objects. It is also ordinal as it is above two, between four and six, below seven. . . There is evidence that we are born with a basic sense of cardinal number (Joyce 2011) which is then further developed through teaching. Ordinality requires access to ordinal long-term memory (where knowledge about ordinal organisation of numbers and information is stored) and the ability to make ordinal judgements about whether or not items are presented in the correct numerical order (Attout & Majerus 2018).

Subitising, magnitude comparison and ordering are core components of number sense and the first steps towards understanding of numeric relationships, development of arithmetical knowledge and reasoning, and development of skills in estimation and manipulation of quantities. An individual who remains dependent upon counting in ones is unlikely to have developed number sense and will have great difficulty with any type of estimation task (Chinn 2017). That individual is also less likely to be able to achieve efficient ways of working. Finesilver (2017) notes that "efficiency" should not be taken to mean knowledge of a particular calculation method – it is dependent upon the individual's ability to select and adapt strategies to suit their current capabilities and the numbers involved, type of task and circumstances under which the task is set.

Domain general deficits and their impact on mathematics

A definition of dyscalculia as a core, domain specific deficit in sense of number makes it both discrete and identifiable within an assessment, but using it introduces the risk that many other individuals with very real mathematics difficulties will go undiagnosed and unsupported. It is thus necessary to also focus attention on other mathematics difficulties which do not fall within the dyscalculia definition, but which are nonetheless potentially disabling.

There is a growing body of evidence that domain general deficits (such as language, memory, processing speed, attention, perceptual reasoning, visual-spatial skills and/or motor coordination) can have an impact upon mathematics learning (Desoete, 2015). A wealth of studies exist which examine relationships between domain general deficits and mathematics learning. For example, the following have identified links between impaired mathematics learning and: verbal and visual working memory (Szucs & Goswami 2013, Geary et al 2017); verbal working memory (Karagiannakis et al 2014, Kroesbergen et al 2015); visual spatial working memory (Mammarella et al 2017, Tosto et al 2014); visual spatial reasoning (Szucs and Goswami 2013, Karagiannakis et al 2014) language skills (Purpura & Ganley 2014, Szucs & Goswami 2013); executive function, attention and inhibitory processes (Wilkey et al 2018, Szucs & Goswami 2013).

These and other studies identify many interrelationships such as:

- Working memory may affect development of number fact knowledge and retrieval; performing mental arithmetic accurately; recognising and discriminating between different mathematics symbols and terminology; being able to translate from one type of number representation to another; remembering and applying rules, formulae and procedures; problem solving.
- Verbal and non-verbal reasoning may contribute to understanding mathematical concepts; understanding multiple steps in complex procedures; following logic steps; and problem solving.
- Visio-spatial abilities may assist with interpreting and applying spatial organization or representations of mathematical objects; understanding and interpreting charts and graphs; aligning numbers accurately, or moving in the correct direction in written calculations; understanding and applying geometry, changes of angle, direction and rotation.
- Executive function is essential for paying attention, selecting relevant versus irrelevant information; inhibiting responses until all relevant information has been processed.

Taking the literature about domain specific and domain general deficits into account, the following two principles offer an evidence-based means of differentiating between dyscalculia and mathematics difficulties arising from other SpLDs.

1. Dyscalculia

The core feature of dyscalculia is a domain specific deficit in sense of number. This manifests as difficulties with subitising, symbolic and non-symbolic magnitude comparison, and ordering (cardinality, ordinality).

The individuals may be able to perform procedures that have been learned by rote (often using single unit counting strategies) but will not understand the underlying concepts or be able to use alternative strategies or adapt to new problems (Chinn 2017). They are unlikely to be able to recognise or check whether their answers are sensible, even when the answer is wildly inaccurate. While a dyscalculic and a dyslexic student may initially present similar incorrect answers to a calculation, the dyslexic student is more likely able to try alternative strategies, while the dyscalculic student is more likely to resort to counting, guessing, or give up. On producing an answer which is wildly inaccurate (e.g. 100 times too large), both students may initially fail to notice this, but on prompting, the dyslexic student is likely to realise that the answer is not sensible, whereas the dyscalculic student is not.

People with dyscalculia will also have a wide range of other mathematics difficulties. Understanding of number is essential to development of arithmetic skills and arithmetic is the first stage of mathematics teaching, so difficulties in this area have a negative impact upon children's subsequent mathematics learning trajectories. Longitudinal research by Jordan et al (2009) reveals a clear link between number competence at kindergarten stage and later arithmetic ability, regardless of socio-economic status, demonstrating the importance of early number competence for setting children's learning trajectories in school mathematics.

Dyscalculia can occur on its own but can also co-occur with other SpLDs (Landerl et al 2009). Statistics on co-occurrence vary widely dependent upon definitions of dyscalculia, but there is consistent evidence that there is some co-occurrence between dyslexia, severe mathematics difficulties, ADHD and Dyspraxia. (Aster and Shalev 2007, Kaufman et al 2013, Landerl et al 2013, Wilson et al 2015, Chinn and Ashcroft, 2017).

2. Other Specific Learning Difficulties

Other SpLDs do not include a deficit in sense of number, but have domain general deficits which can include language, working memory, processing speed, attention, perceptual reasoning, visual-spatial skills and/or motor coordination. There is substantial evidence that these general domain deficits can potentially affect all types of learning, including mathematics.

There are concerns within the mathematics community that problems with mathematic arising from SpLDs are often less well understood than co-occurring literacy difficulties. This is partly because there has been far more research into dyslexia (and in particular, the effect of dyslexia upon literacy) than any of the other SpLDs.

It is important to redress this imbalance by ensuring that difficulties with mathematics are explored appropriately within diagnostic assessments. Where those difficulties are found to be similar to, or greater than the difficulties with literacy, assessors could note that *the specific learning difficulty has a clear and specific impact upon mathematics.*

An individual with a specific learning difficulty may, therefore, present with co-occurring difficulties in literacy and mathematics. Development of arithmetical skills calls upon a wide range of domain general abilities and the more complex the calculations become, the greater the impact of any domain general deficits will be (Cowan & Powell 2013). This has a cascade effect upon mathematics skills.

However, it should not be assumed that general domain deficits will inevitably lead to weaker performance in mathematics. As with literacy, appropriate training, extensive practice, and compensatory use of other strengths and strategies can enable an individual to perform well in mathematics despite specific weaknesses in cognitive processing.

It is also necessary to bear in mind that tests which use numbers for measuring cognitive processes (such as digit span tests or rapid number naming) may reflect weak development of number skills or number sense (Kaufman et al 2013). Where a sense of number or number skills are suspected to be underdeveloped, it makes sense to administer additional non-number based tests of cognitive processing and compare the results.

Dyscalculia Definition

(Agreed by both the SASC/STEC and BDA Working Groups on Dyscalculia)

Dyscalculia is a specific and persistent difficulty in understanding numbers which can lead to a diverse range of difficulties with mathematics. It will be unexpected in relation to age, level of education and experience and occurs across all ages and abilities.

Mathematics difficulties are best thought of as a continuum, not a distinct category, and they have many causal factors. Dyscalculia falls at one end of the spectrum and will be distinguishable from other mathematics issues due to the severity of *difficulties with number sense, including subitising, symbolic and non-symbolic magnitude comparison, and ordering. It can occur singly but can also co-occur with other specific learning difficulties, mathematics anxiety and medical conditions.*

What should be included in an assessment of difficulties with mathematics?

Diagnostic assessment of difficulties in mathematics should form part of a holistic assessment designed to explore **the full range of SpLDs** and **cognitive, medical and environmental factors** that may be contributing to difficulties with learning (APA 2013, Kaufman et al 2013). They should explore and evaluate response accuracy, speed, and strategies, and recognise that areas of specific difficulty are possible within an individual's mathematics profile even if the overall mathematics test scores may appear appropriate (Kaufman et al 2013).

A holistic, in-depth assessment of mathematics difficulties would, therefore, ideally include the following:

- A framework for a thorough and appropriate history taking which covers mathematics, literacy and wider barriers to learning. Although standardised tests are useful for maintaining professional standards, difficulties with mathematics should not be diagnosed purely on the basis of scores achieved in mathematics attainment tests as children can attain a low score in mathematics for a variety of reasons. Kaufman and Von Aster (2012) highlight the need for a thorough investigation in the individual's background and history of mathematics difficulties and provide a helpful list of questions for history-taking where dyscalculia or mathematics difficulties are suspected. This history-taking should also include questions about attitudes and motivation, including avoidance of mathematics activities, and mathematics anxiety
- Tests of verbal reasoning, visual and visual-spatial reasoning, and cognitive processing (such as memory, phonological processing, processing speeds, and visual spatial sequential skills) in order to identify strengths and domain general deficits within the cognitive profile.
- Tests of literacy and mathematics skills in order to identify strengths and weaknesses within the attainment profile. The assessor will use professional expertise and discretion to decide which areas of literacy and/or mathematical skills need to be explored, dependent upon the range of difficulties reported/suspected by the individual and/or parents/teachers.
- Informal, qualitative tests of understanding of number that use symbolic and non-symbolic magnitude comparison, subitising, and estimation. These should include physical manipulatives (e.g. cubes, counters) to explore concept of number through physical as well as graphic and symbolic representations. This could include screeners designed to explore number sense.
- Standardised measures of
    - Arithmetic (+, -, x, ÷). Timed and untimed to establish what difference time pressure makes upon performance. Including simple sums (to explore speed

and accuracy of recall of mathematics facts) and more complex sums to evaluate recall and application of such procedures.
- Mathematics reasoning and problem solving, including word problems to explore whether the difficulties relate to sense of number, mathematical terminology, or language more generally.

Qualitative analysis of performance within these tests:
- Analysis of the individual's pattern of errors,
- Observation and questioning about strategies used
- Observation of motivation, determination, perseverance, impulse inhibition, attention, and which tasks were avoided,
- Conceptual understanding of any procedures used
- Use of concrete materials and visual representations – to evaluate to what extent an individual understands basic mathematical concepts, and to explore any differences between what an individual can achieve with standard symbolic notation, and with nonstandard representational strategies (e.g. modelling, drawing, Trott 2018). For example, Finesilver (2017a) sets out a framework for microanalysis of learners' early understanding of multiplication using array tasks and blocks.

When choosing tests, it is important to consider how these will affect the overall length of the assessment and the individual's ability to sustain effort and concentration over a long period of time.

Recommendations for interventions and reasonable adjustments should be clearly linked to

- The individual's difficulties reported in the background information and evidenced in the assessor's quantitative and/or qualitative analysis of performance in tests.
- The individual's needs within the classroom, course or job. Wherever possible recommendations should be developed collaboratively with relevant mathematics specialists in the individual's school, course or workplace.

Reasonable adjustments should be appropriately targeted to address the need without potentially giving the individual an unfair advantage. Assessors should bear in mind that adjustments such as use of a calculator or provision of rest breaks can sometimes target the need more effectively than additional time.

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